Nuclear-Thermal vs Chemical Drives for Super-Orbital Velocities

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MANY experts in the field of astronautics seem to think that, in the long run, the nuclear drive will be universally used and will supersede the chemical drive. To some extent, this is probably true. However, there can be little doubt that there will always be applications and missions for which the chemical drive will be more practical.

Obviously, a marked difference exists between requirements for propulsion units for rockets operating at less than orbital velocity and for rockets operating at velocities higher than that. In the first case, a unit is needed with high thrust-to-weight ratio, and the problem is to build large propulsion units. In the second case, the thrust-to-weight ratio may be very low, and the engines needed are much smaller. High specific impulse is particularly desirable for the superorbital.

We shall confine ourselves here to a discussion of the superorbital vehicle because in this case it is possible to arrive at a quite definite and clear-cut result. For the suborbital vehicle, comparison is more complicated; moreover, the nuclear drive is expected to be used in upper rocket stages first.

Since, in the last stages, there is a premium on high specific impulse and on good vector control, it is clear that the high-energy liquid-propellant engine is the best among the chemical engines available. Compared to the nuclear-thermal engine, however, its specific impulse is still much lower (about 400 sec). On the other hand, the engine weight is low.

The nuclear drive may be either nuclear-thermal or nuclear-electric. The latter term comprises all sorts of electrothermal, electrostatic, and magnetohydrodynamic propulsion units. It is generally agreed that the best powerplant for these devices is the nuclear-electric plant, so that a nuclear pile has to be used in any case, and the question is only whether this pile should heat the propellant directly (nuclear-thermal drive) or should be used to generate electric power to accelerate or heat the exhaust jet (nuclear-electric drive). Only the nuclear-thermal drive, which seems to be closer to realization, will be considered here for comparison to the chemical drive. The minimum practical weight of a nuclear-thermal engine is estimated at approximately 1000 lb for a thrust of 5000 lb.[1] The specific impulse can be made as high as 800 sec. On the basis of these data, a comparison to the chemical drive can be made.

This is done by establishing three conditions that must be satisfied for two "equivalent" rockets having nuclear and chemical drives, namely, 1) since rockets like these will be the last stages of multistage carrier vehicles, we have to stipulate that the initial weight W is to be the same for both of them; 2) the velocity increase ΔV to be imparted to the rockets is to be the same; and 3) the two rockets shall carry the same payload L. In the following, the subscript n will be used for the nuclear rocket and c for the chemical rocket. Let P be the weight of propellant, I the specific impulse, and g the acceleration of gravity. Then we can express forementioned conditions 1 and 2 in the form

$$\Delta V = gI_n \ln \frac{W}{W - P_n} = gI_c \ln \frac{W}{W - P_c} \tag{1}$$

where we assume that $I_n = 2I_c = 800$ sec. Further, we introduce the new variable

$$z = P_n/W = 1 - \exp(-\Delta V/gI_n) \tag{2}$$

Then, one finds from Eq. (1)

$$P_c/W = z(2 - z) \tag{3}$$

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For further discussion we shall have to introduce a suitable assumption concerning the weights of the rockets. For the nuclear rocket, up to a total initial weight of about twice the engine thrust (i.e., up to 10,000 lb), the minimum-size engine is chosen so that the engine weight, together with the weight of the guidance and control system, can be considered constant, i.e., independent of rocket size. Beyond that limit, velocity losses due to gravity begin to become significant, so that a larger engine with a higher thrust should be used. Constant engine weight is assumed also for the chemical rocket (although the engine could be smaller for smaller rockets) because this will not affect the equivalence conditions [Eqs. (6) and (7) below]. Another part of the total weight, namely, the weight of the tanks plus supporting structure, can be considered as being proportional to the propellant weight if the rocket is designed for its particular mission, i.e., if the tanks are completely filled (allowing for ullage) and their wall thickness is the minimum required for the inside pressure.

Then the total initial weight of the nuclear rocket will be

$$W_n = P_n + A_n + B_nP_n + L = W_nz(1 + B_n) + A_n + L \tag{4}$$

and the total initial weight of the chemical rocket

$$W_c = W_cz(2 - z)(1 + B_c) + A_c + L \tag{5}$$

Letting $W_n = W_c = W_e$, one finds the "equivalent" initial weight W_e of the two rockets:

$$W_e = \alpha/z(\beta - z) \tag{6}$$

and the corresponding "equivalent" payload L_e:

$$L_e = W_e[1 - z(1 + B_n)] - A_n \tag{7}$$

where

$$\alpha = (A_n - A_c)/(1 + B_c)$$
$$\beta = 1 - (B_n - B_c)/(1 + B_c)$$

The weight of the nuclear engine, as we have seen, is considerably greater than that of the chemical engine. Hence, $A_n > A_c$, and $\alpha > 0$. The tank weight is also greater for the nuclear engine than for the chemical engine because of the greater bulk specific gravity of the chemical propellants. Hence $B_n > B_c$, and $\beta < 1$. Since z is a known function of ΔV [see Eq. (2)] we have a value W_e and a value L_e for each ΔV. The velocity increase is specified according to the mission. The initial weight W is often directly known as the carrying capacity of the available booster vehicle. If then W is smaller than W_e, the chemical rocket is better than the nuclear rocket because it can carry a heavier payload for the same mission. If W is larger than W_e, the nuclear rocket is "potentially" superior, i.e., as far as payload is concerned. This does not necessarily mean that the nuclear drive is really better. There are other factors to be considered, such as complexity of the system, reliability, radiation hazards, and handling difficulties, which may still outweigh the advantage of higher payload of the nuclear rocket, at least for an initial weight not too high above W_e.

However, sometimes it is the payload that is specified by the mission (in addition to ΔV) and that should be taken as a basis for comparison. If for a given velocity increase the specified payload L is smaller than L_e, the chemical rocket is better, because its initial weight will be smaller than that of the nuclear rocket carrying the same payload. If the payload is greater than L_e, the nuclear rocket is "potentially" superior, i.e., as far as initial weight is concerned.

Of course, the two methods of comparison always give the same result because we started from the assumption that, for "equivalent" rockets, both initial weight and payload must be the same.

Equivalent initial weight and payload as determined by Eqs. (6) and (7) can be plotted in a graph against the velocity increase (Fig. 1). For the rockets of moderate size which are compared here, a pressurized tank system should be used

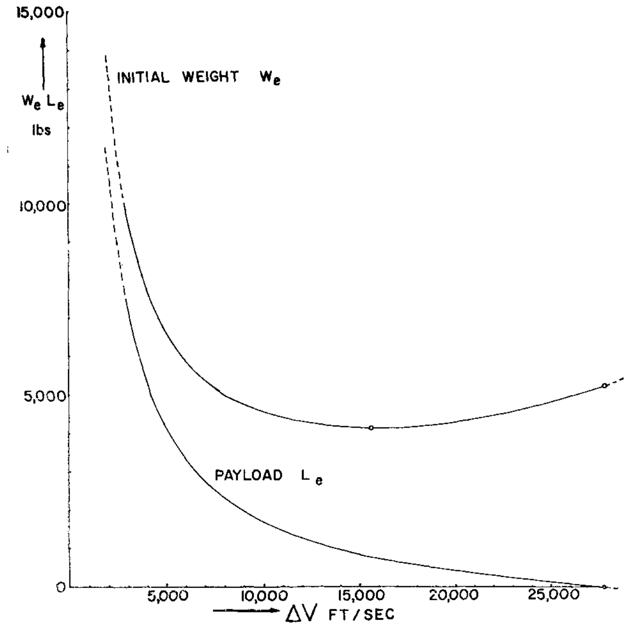

Fig. 1 Equivalence of nuclear and chemical drives.

rather than a turbopump for feeding propellants to the engine. Under this assumption, the following values were estimated for the constants: $A_n = 1100$ lb, $A_e = 150$ lb, $B_n = 0.20$, and $B_c = 0.10$. This gives $\alpha = 863.636$ lb, and $\beta = 0.90909$.

The curves separate the area of superiority of the chemical drive from the area of potential superiority of the nuclear drive in terms of ΔV and W or ΔV and L.

The curve for W_e has a minimum W_1 for $z = \frac{1}{2}\beta$:

$$W_1 = 4\alpha/\beta^2 = 4180 \text{ lb}$$

The corresponding velocity increase is $\Delta V_1 = 15614$ fps, so for rockets having an initial weight up to W_1, the chemical drive will always be better. For a heavier rocket, the velocity increase has to be considered. For an increase smaller than ΔV_1, W_e becomes larger and becomes infinite for $\Delta V = 0$. In other words, for a very small velocity increase the chemical drive is always better. This is obvious because then the weights of either nuclear or chemical propellants needed will be very small and there is nothing to offset the difference in engine weights. For values of ΔV larger than ΔV_1, W_e increases again, because of increasing propellant weight.

The curves break off for $\Delta V_2 = 27{,}635$ fps, at which the equivalent payload becomes zero. That does not mean that a single-stage rocket cannot carry any payload for a velocity increase larger than ΔV_2, but then the initial weight has to be larger than W_e; hence the nuclear rocket is better.

A payload of 200 lb of instruments will be large enough for most scientific measurements. With this value of L_e one finds a velocity increase of 23,676 fps. This is sufficient to send a vehicle from a 300-mile parking orbit around Earth to the vicinity of Saturn or well within the orbit of Mercury. This shows that for most unmanned missions in the solar system the chemical drive will be better than the nuclear-thermal and probably also the nuclear-electric drive that has a still higher engine weight.

The "equivalent" curves are, of course, dependent on the choice of the constants I_n, I_c, A_n, A_c, B_n, and B_c which affects the numerical values of W_1, ΔV_1, and ΔV_2. However, the minimum equivalent weight W_1 is found to vary only slightly within the practical limits of these constants.

Reference

[1] Penner, S. S., *Advanced Propulsion Techniques* (Pergamon Press, New York, 1961), p. 54.

Buckling of a Slender Prismatic Rod at High Impact Velocity

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THIS paper is addressed to the following question: For given high-velocity impact conditions, can column buckling occur in a slender prismatic rod penetrating a target? Many analyses have been carried out for impact buckling of elastic rods.[1–4] Most of these have dealt with column action in rapid compression tests, rather than with the simpler question of whether or not buckling can initiate. The latter topic has been treated,[4] but the assumption of an elastic material limits the impact velocity v_0 to low values, because it is equal to $\epsilon_e c_e$, where ϵ_e is the elastic-limit strain, and c_e is the acoustic velocity of the rod. For most materials, $v_0 < 200$ fps for elastic conditions. Some plastic-range impact-buckling studies have been made,[5] but, apparently, none have been for high-velocity impact conditions, when the material obeys a nonlinear equation of state.

Here an analysis for an arbitrary material-behavior regime is carried out and applied to the high-velocity regime, using available data on rod penetration and mechanical equations of state.

Analysis

A slender rod with a uniform, compact cross section is struck axially at one end with a relative velocity v_0 at time $t = 0$. The impact force is not removed immediately but remains constant during a given time interval. A compressive pulse propagates along the rod at some velocity c (elastic, plastic, or shock, as the case may be) and has traveled a distance ct into the rod at time t. Assuming that penetration takes place at constant deceleration from the initial relative velocity v_0 to a final velocity $(v_0 - \Delta v)$ where Δv is the total velocity change for the given rod-target combination, the relative velocity during penetration is given by

$$v_p = v_0 - (t\Delta v/t_p) \tag{1}$$

where t_p is the time required for complete penetration.

The combination of rod penetration and disintegration of the rod end is assumed to be a steady-state process, and thus the instantaneous velocity of rod-end disintegration is $v_d = Av_p$, where A is a constant. Thus, the physical length subjected to axial compression at time t is given by

$$L_p = (c - Av_0)t + (A\Delta v t^2/2t_p) \tag{2}$$

The generalized Euler equation is appropriate for buckling of a slender rod[6]:

$$\sigma_{cr} = E_t(\pi k_{cr}/L)^2 \tag{3}$$

where σ_{cr} is the critical axial load per unit area at which buckling can occur; E_t is the tangent modulus (i.e., the slope of the stress-strain curve at stress σ_{cr}), k_{cr} is the critical minimum radius of gyration of the rod cross section (square root of the ratio of minimum centroidal moment of inertia to the cross-sectional area); and L is the equivalent buckling length

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